10/12/12. Maths.

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FIRST COMMON PRE-BOARD EXAMINATION 2012-13

MATHEMATICS-CLASS XII

TIME: 3 hrs **MAX MARKS:100**

General Instructions:

1.All questions are compulsory

2.The question paper consists of 29 questions divided into three sections A,B and C.Section A comprises of 10 questions of 1 mark each,Section B comprises of 12 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.

3.All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4.There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5.Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. If $f: R \rightarrow R$ be defined by $f(x) = (3 - x^3)^{\frac{1}{3}}$. Find $f \circ f(x)$.

2. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.

3. Find the value of x if $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

4. Evaluate $\int \sec^2(7 - 4x)\,dx$

5. Write A^{-1} in terms of A if , $A=\begin{bmatrix} -3 & -1 \\ 4 & 3 \end{bmatrix}$

6.Write the value of the following integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5 x\,dx$

7. A is a square matrix of order 3 and |A| = 9 . Write the value of |adj A|.

8. Write the distance of the following plane from the origin $2x - y + 2z + 1 = 0$

9. Write a vector of magnitude 8 units in the direction of vector $-4\hat{i}+2\hat{j}+4\hat{k}$.

10. Find λ if $(2\hat{i}+6\hat{j}+14\hat{k}) \times (\hat{i}-\lambda\hat{j}+7\hat{k}) = 0$.

SECTION- B

11. The length x of a rectangle is decreasing at the rate of 5cm/min and the width y is increasing at the rate of 4cm/min. When x=8cm and y=6. Find the rate of change of

(a) the perimeter

(b) the area of the rectangle

OR

Find the intervals in which the function f given by $f(x) = \sin x + \cos x 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

12. **Showthat f(x)=** $\begin{cases} \frac{sin3x}{tan2x}, if\ x > 0 \\ \frac{3}{2}, if\ x = 0 \\ 3\frac{(e^x-1)}{e^{2x}-1}, \quad if\ x < 0 \end{cases}$ **is continuous at x=0**

13. Show that the function f: $R \to R$ defined by f(x) = $\frac{2x-1}{3}$, x∈ R, is one –one and onto function .Also find the inverse of the function f.

14. Evaluate $\int \frac{dx}{\sqrt{5-4x-2x^2}}$ OREvaluate $\int x \sin^{-1} x\, dx$

15. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

16. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, What is the probability that a candidate would get four or more correct answers just by guessing?

Is guess method a right method? What other method would you suggest for getting correct answer? Write a suitable value that you can think of.

17. Using properties of determinants, show that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

18. Solve the following differential equations

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

19. Solve the following differential equation

$$\cos^2 x\frac{dy}{dx} + y = \tan x$$

OR

Solve the differential equation : $\frac{dy}{dx} = 3yCotx + Sin2x$,

given that y=2,when $x = \frac{\pi}{2}$.

20. Find the shortest distance between the following two lines

$$\vec{r} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (\lambda+1)\hat{k}$$
$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

21. Prove the following

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{\pi}{2}, x\epsilon\left(0, \frac{\pi}{4}\right)$$

OR

Solve for x , $2\tan^{-1}(cosx) = \tan^{-1}(2\,cosec\,x)$

22. .Let $\bar{a} = \bar{i} + 4\bar{j} + 2\bar{k}$ $\bar{b} = 3\bar{i} - 2\bar{j} + 7\bar{k}$ $\bar{c} = 2\bar{i} - \bar{j} + 4\bar{k}$ Find a vector $\bar{d}$ which is perpendicular to both $\bar{a}$ and $\bar{b}$ and $\vec{c}.\vec{d} = 15$

SECTION-C

23. Find the equation of the plane determined by points A(3,-1,2), B(5,2,4), and C(-1,-1,6) also find the distance of the point P(6,5,9) from the plane.

24.Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

OR

Using integration of the area of the triangle ABC whose coordinates of whose vertices are A(4,1)B(6,6)and C(8,4)

25. Evaluate $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$